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Fast cavity auto-tuning systems for hydrogen masers

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Résumé. — On considère un système d'accord automatique de la cavité résonnante de masers à hydrogène. On montre qu'il peut être associé avec un maser oscillateur. Il assure un contrôle serré de la fréquence de résonance de la cavité, avec une constante de temps courte. Ceci est favorable à l'amélioration de la stabilité de fréquence à long terme de cet étalon de fréquence. Il existe des conditions de fonctionnement qui ne perturbent pas la stabilité de fréquence à long terme ultime qu'il est possible d'atteindre. On montre que la fréquence de la résonance de la cavité peut être aisément ajustée de telle sorte que le déplacement de fréquence par échange de spins soit éliminé, aussi bien qu'avec les autres méthodes connues d'accord de cavité.

Abstract. — An auto-tuning system for the microwave cavity of hydrogen masers is considered. It is shown that it can be applied to oscillating masers. A tight control of the cavity resonant frequency is insured, with a short time constant. This is favourable to the improvement of the long term frequency stability of this frequency standard. Operating conditions exist which do not perturb the ultimately achievable long term frequency stability of the standard. It is shown that the microwave cavity resonant frequency can be easily adjusted so that the spin exchange frequency shift is eliminated as well as with known other cavity tuning methods.

1. **Introduction.** — In the hydrogen maser [1], the cavity pulling effect, although very small, limits the long term frequency stability of this frequency standard. The related frequency deviation, Δf_0, of the oscillation frequency is given by :

$$\Delta f_0 = \frac{Q_c}{Q_l} \Delta f_c \qquad (1)$$

where Δf_c is the cavity mistuning, Q_c and Q_l are the quality factors of the loaded microwave cavity and the atomic line, respectively. One has, typically :

$$Q_c = 3.5 \times 10^4 \quad \text{and} \quad Q_l = 1 \times 10^9 .$$

For instance, a long term fractional frequency stability of 10^{-14} requires that the cavity resonant frequency is controlled within 0.4 Hz, around 1.42 GHz.

Two methods have been proposed to probe the cavity mistuning. The first one [1-2] is founded on equation (1) and consists in the observation of the oscillation frequency change while the atomic line quality factor is modulated. It has been widely implemented [3]. The second one is based on a correlation between the phase and the amplitude of oscillation [4].

Both methods are conceptually satisfactory as they are able to eliminate cavity mistunings through the monitoring of the frequency or the phase of oscillation, two quantities of prime interest in frequency standards. However, they share in common two drawbacks : i) fluctuations in the frequency or the phase of oscillation limit the efficiency of the control of the cavity resonant frequency and ii) the time constant of the cavity control is very large, of the order of one hour or more [5].

On the other hand, a more traditional means to tune the microwave cavity has proved to be very useful in recent developments of passively operated hydrogen masers [6], which are studied for the purpose of size reduction. We show here that this method of cavity tuning can be applied to an oscillating maser (actively operated) and that it provides significant advantages.

2. **Control of the resonant frequency of the microwave cavity.** — 2.1 PRINCIPLE OF THE METHOD. — It is described in figures 1 and 2. A frequency modulated microwave signal is transmitted through the cavity ([1]). For sake of simplicity, we assume a square-wave modulation. One takes care that the power of the input signal is the same at both frequencies. The amplitude a of the transmitted signal is also modulated, and an error signal is provided when $\omega_r \neq \omega_c$, where

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([1]) The reflected signal might be considered as well.

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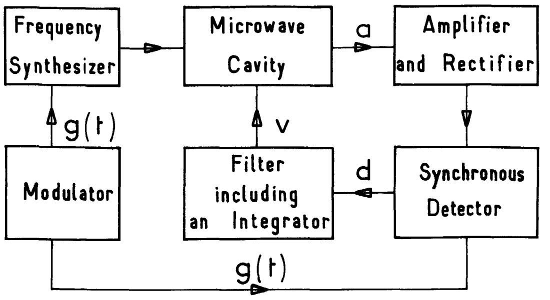

Fig. 1. — Block diagram of the considered cavity tuning system.

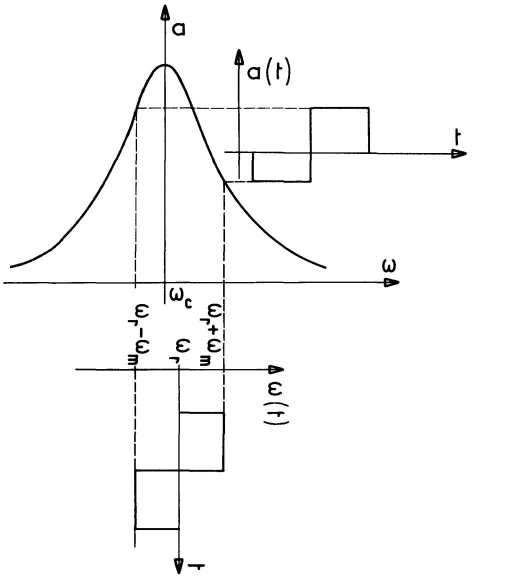

Fig. 2. — Amplitude response of the microwave cavity to the square-wave frequency modulated excitation.

ω_r is the mean value of the reference angular frequency and ω_c is the cavity resonant angular frequency.

The angular frequency $\omega(t)$ of the applied signal is given by

$$\omega(t) = \omega_r + \omega_m g(t) \qquad (2)$$

where ω_m is the angular frequency modulation depth and $g(t)$ is a function with period T such as :

$$g(t) = 1 \quad \text{for} \quad 0 < t < T/2$$
$$g(t) = -1 \quad \text{for} \quad T/2 < t < T . \qquad (3)$$

The value of T should be larger that the microwave cavity time constant τ_c. We have $\tau_c \simeq 10$ μs in a hydrogen maser. A convenient value of T is then $T \simeq 10^{-3}$ s.

The amplitude $a(t)$ is amplified and linearly detected. It is then multiplied by $g(t)$ in a synchronous detector. It can be seen that after a low pass filter, the useful, low frequency component, of the signal obtained is d_1 given by :

$$d_1 = K(\omega_r - \omega_c) \left.\frac{\partial a}{\partial \omega}\right|_{\omega_c + \omega_m} \qquad (4)$$

where K is the gain of the chain composed of the amplifier, the rectifier and the synchronous detector. It has been assumed that : i) the quantity $(\omega_r - \omega_c)$ is much smaller than the cavity line-width and is a slowly varying function of time and ii) the cavity resonance pattern is an even function of ω.

The thermal noise in the microwave cavity, as well as the amplifier noise, give the contribution d_2 to the signal observed after the synchronous detector. If $\delta a(t)$ denotes the related amplitude fluctuations, as measured at the input of the amplifier, we have :

$$d_2 = K\, \delta a(t).g(t) = K d_2' . \qquad (5)$$

At the output of the filter, with transfer function $F(p)$, we obtain the voltage v given by :

$$v(p) = [d_1(p) + d_2(p)]\, F(p) \qquad (6)$$

where the notation $x(p)$ means the Laplace transform of $x(t)$.

This voltage is applied to a varactor which corrects the cavity resonant frequency. The servo-loop equation is then :

$$\omega_c^s(p) = C_c\, v(p) + \omega_c^o(p) \qquad (7)$$

where the subscript c refers to the cavity resonant frequency. The upper scripts s and o characterize the considered quantity when the cavity is under the control of the servo-loop, or out of control (open-loop), respectively. C_c is a constant.

For the purpose of an easy derivation of useful properties, we assume that the filter simply consists of an integrator with transfer function $F(p) = 1/p$. We then introduce the time constant $\mathcal{T}$ defined by :

$$C_c\, K F(p) \left.\frac{\partial a}{\partial \omega}\right|_{(\omega_c + \omega_m)} = \frac{1}{p\mathcal{T}} . \qquad (8)$$

For small variations of the angular frequencies ω_c^c, ω_c^o and ω_r, we have, from equations (4), (6), (7) and (8) :

$$\frac{\Delta\omega_c^s(p)}{\omega_c} = \frac{p\mathcal{T}}{1 + p\mathcal{T}} \frac{\Delta\omega_c^o(p)}{\omega_c} + \frac{1}{1 + p\mathcal{T}} \frac{\Delta\omega_r}{\omega_c} + \frac{1}{1 + p\mathcal{T}} \frac{1}{\left.\dfrac{\omega_c}{a_m}\dfrac{\partial a}{\partial \omega}\right|_{(\omega_c + \omega_m)}} \frac{1}{a_m} d_2'(p) \qquad (9)$$

where a_m is the amplitude of the cavity response for $\omega = \omega_c + \omega_m$.

It is clear that $\mathcal{T}$ is the time constant of the servo-loop response. As it is very well known, a small value of $\mathcal{T}$ gives a tight control of the cavity resonant frequency.

In the last term of equation (9), the effect of amplitude noise is minimized when the quantity

$$\frac{1}{a_m} \left.\frac{\partial a}{\partial \omega}\right|_{(\omega_c + \omega_m)}$$

is a maximum. This is achieved for :

$$\omega_m = \omega_c / 2\, Q_c \qquad (10)$$

where Q_c is the quality factor of the loaded microwave cavity. One has $\omega_m / 2\pi \simeq 20$ kHz for

$$\omega_c / 2\pi = 1.42\ \text{GHz} \quad \text{and} \quad Q_c = 35\,000\,.$$

It is assumed in the following that ω_m is adjusted to its optimum value. We then have :

$$\frac{\omega_c}{a_m} \left.\frac{\partial a}{\partial \omega}\right|_{(\omega_c + \omega_m)} = Q_c\,. \qquad (11)$$

2.2 POWER SPECTRAL DENSITY OF CAVITY FREQUENCY FLUCTUATIONS. — It can be seen that the power spectral density (P.S.D.) ([2]) of $d_2'(t)$, for angular Fourier frequencies Ω [7, 8] much smaller than $1/T$ is $S_{d_2'}(\Omega)$ given by [9]

$$S_{d_2'}(\Omega) = \frac{8}{\pi^2} S_{\delta a}\left(\Omega = \frac{1}{T}\right) \qquad (12)$$

where $S_{\delta a}$ is the P.S.D. of noise fluctuations $\delta a(t)$.

The P.S.D. of fractional angular frequency fluctuations, $y = \Delta\omega/\omega_c$ [10], of the servo-controlled cavity is then given by :

$$S_y^s(\Omega) = \frac{\Omega^2 \mathcal{T}^2}{1 + \Omega^2 \mathcal{T}^2} S_y^o(\Omega) + \frac{1}{1 + \Omega^2 \mathcal{T}^2} S_y^r(\Omega) + \\ + \frac{8}{\pi^2 Q_c^2} \frac{1}{1 + \Omega^2 \mathcal{T}^2} S_{\delta a/a_m}\left(\Omega = \frac{1}{T}\right) \qquad (13)$$

where $S_y^o(\Omega)$ is the P.S.D. of the fractional angular frequency fluctuations of the cavity which needs to be corrected. $S_y^r(\Omega)$ is the P.S.D. of the fractional angular frequency fluctuations of the reference signal and $S_{\delta a/a_m}$ is the P.S.D. of fractional amplitude fluctuations.

2.3 EFFECT OF FREQUENCY NOISES. — The first term of the right hand side of equation (13) shows that the low frequency components of the frequency fluctuations of the microwave cavity are more efficiently washed out when the value of $\mathcal{T}$ is small. This is the condition to be fulfilled to ensure a good long term frequency stability of the frequency standard. The choice of the value of $\mathcal{T}$ can then be made, knowing that $\mathcal{T}$ must be larger than the modulation period T. The value $\mathcal{T} \simeq 1$ s looks convenient.

The second term describes the effect of the frequency noise of the reference signal. Its low frequency components perturb the resonant frequency of the controlled microwave cavity. The reference source must then show an excellent long term frequency stability. This is conveniently achieved when the reference signal is frequency synthesized from the maser oscillation itself as described in section 4.

2.4 EFFECT OF AMPLITUDE NOISE. — It is given by the last term of equation (13) and it determines the long term frequency stability. For $\Omega \ll 1/T$, one has :

$$S_y^s(\Omega) = \frac{8}{\pi^2 Q_c^2} S_{\delta a/a_m}\left(\Omega = \frac{1}{T}\right). \qquad (14)$$

It can be shown that we have [11, 12] :

$$S_{\delta a/a_m}\left(\Omega = \frac{1}{T}\right) = \frac{4\,kT}{P_c}\left[1 + (F - 1)\frac{Q_{ext}}{Q_c}\right] \qquad (15)$$

where k is Boltzman constant, T is the absolute temperature of the cavity, P_c is the microwave power dissipated in the microwave cavity at frequency $\omega_c \pm \omega_m$, F is the noise factor of the amplifier, Q_{ext} and Q_c are the external and loaded cavity quality factors, respectively.

It results from equations (1), (14) and (15) that for $\Omega \ll 1/T$, the P.S.D. of fractional frequency fluctuations of the maser oscillation, which is induced by the cavity control servo-loop is $S_y'(\Omega)$ given by :

$$S_y'(\Omega) = \frac{32}{\pi^2} \frac{1}{Q_l^2} \frac{kT}{P_c}\left[1 + (F - 1)\frac{Q_{ext}}{Q_c}\right]. \qquad (16)$$

The spurious amplitude noise will not affect the efficiency of the cavity tuning method, and will allow to reach the ultimately achievable long term frequency stability, if $S_y'(\Omega)$ is smaller than $S_y(\Omega)$, the P.S.D. of frequency fluctuations which result of the effect of thermal noise on the maser oscillation itself. We have [12, 13] :

$$S_y(\Omega) = \frac{1}{Q_l^2} \frac{kT}{P_0} \qquad (17)$$

where P_0 is the power delivered by atoms to the microwave cavity. This comparison sets a lower limit to the power P_c which is injected into the microwave cavity for the purpose of frequency control. It comes :

$$\frac{P_c}{P_0} \gg \frac{32}{\pi^2}\left[1 + (F - 1)\frac{Q_{ext}}{Q_c}\right]. \qquad (18)$$

For the following typical values : $F = 2$, $Q_{ext} = 8 \times 10^4$ and $Q_c = 3.5 \times 10^4$, one has : $P_c/P_0 \gg 10$. P_0 being of the order of 10^{-12} W, a convenient value of P_c is 10^{-10} W.

3. **Spurious frequency shifts introduced by the method are negligible.** — 3.1 POWER INJECTED INTO THE OSCILLATOR. — Injection of power into an oscillator pulls the oscillation frequency towards the injected one. Synchronization may even occur. Here, the sepa-

([2]) Throughout this paper, only one-sided power spectral densities are considered.

ration between the injected frequency and the free oscillation frequency amounts to $\pm \omega_m$. The synchronization condition is then far from being satisfied for $P_c/P_0 \simeq 10^2$. However, a frequency shift Δf occurs [14]. As shown in Appendix, it is given by :

$$\frac{\Delta f}{f_1} = -\frac{1}{8 Q_1^2} \frac{\omega_1}{\omega - \omega_1} \frac{P_c}{P_0} \tag{19}$$

where $f_1 = \omega_1/2\pi$ is the free oscillation frequency. For $Q_1 = 10^9, f_1 = 1.42$ GHz, $\omega - \omega_1 = 2\pi \times 20$ kHz and $P_c/P_0 = 10^2$, one has $|\Delta f/f_1| = 9 \times 10^{-13}$.

The maser oscillation frequency will then be square-wave frequency modulated, with the period T and with a fractional amplitude of 9×10^{-13}.

However, for $\tau = T \simeq 10^{-3}$ s, random fractional fluctuations of the oscillation frequency are much larger, of the order of 10^{-10}.

Furthermore, the output signal of the frequency standard is delivered by a quartz crystal oscillator which is phase-locked to the maser, with a time constant of about 0.1 s. Fast modulation of the maser frequency, with a period of 10^{-3} s, is then filtered out by the phase loop [15] and does not appear on the output signal.

The considered spurious frequency modulation has thus a quite negligible effect.

3.2 Virtual transitions. — The resonant frequency f_0 of two level quantum systems is changed when a perturbation at the frequency $f \neq f_0$ is applied [16]. This effect can be interpreted in terms of virtual transitions [17]. The related frequency shift $\Delta f'$ is given by :

$$\frac{\Delta f'}{f_0} = -\frac{b^2}{2\omega_0(\omega - \omega_0)} \tag{20}$$

where $\omega_0 = 2\pi f_0$. Here, ω is the angular frequency of the microwave signal injected into the cavity. Thus, we have $|\omega - \omega_0| = \omega_m$. The quantity b is the Rabi angular frequency of atoms submitted to the excitation with frequency ω. We have $b^2 = P_c b_0^2/P_0$ where b_0 is the Rabi angular frequency of atoms in the field created by the maser oscillation, a quantity which can be measured [4]. For $P_c = 10^{-10}$ W, we have $b^2 \simeq 1.5 \times 10^3$ rad^2.s^{-2}. Taking $\omega_m/2\pi = 20$ kHz, and $\omega_0/2\pi = 1.42$ GHz, we get $|\Delta f'/f_0| = 7 \times 10^{-13}$. We find the same order of magnitude as for the effect considered in section 3.1, and the same conclusion applies.

4. **Possible realization of the method.** — It is depicted in figure 3. A quartz crystal oscillator is phase locked to a hydrogen maser [18] using well known techniques. It provides the output signal of the frequency standard, without any perturbation by the cavity auto-tuning process, as shown above.

This quartz crystal oscillator feeds a frequency

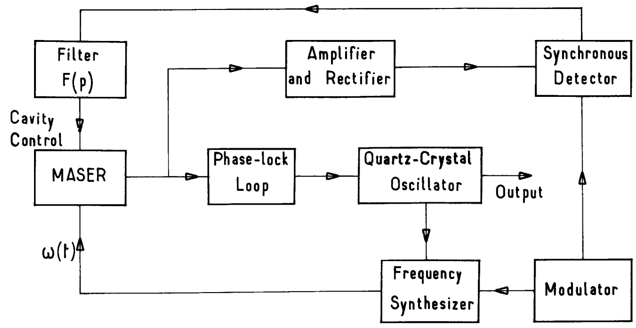

Fig. 3. — Block diagram of the fast auto-tuning system for hydrogen masers.

synthesizer. It is frequency modulated, so as to give the frequency $\omega(t)/2\pi$ defined by equation (2). The signal transmitted by the microwave cavity is amplified and rectified. The synchronous detector then gives the error signal $\hat{d_1}$ which is filtered and then applied to a varactor coupled to the microwave cavity. The power level of the signal injected into the cavity is easily adjusted by comparing the levels of the maser oscillation and of the cavity-transmitted signal. The desired value of the time constant $\mathcal{T}$, of the order of 1 s, is obtained by a proper choice of the parameters of the cavity tuning loop, according to equation (8).

The reference frequency being synthesized from the maser oscillation, frequency fluctuations of the cavity tuning frequency are closely correlated to that of the maser signal, for observation times larger than $\mathcal{T}$. These fluctuations react on the maser oscillation frequency, as shown by equation (1), but only negligeably thanks to the factor Q_c/Q_1 which is of the order of 3.5×10^{-5}. This feedback is then very weak and does not change the conclusions of sections 2 and 3.

5. **Elimination of the spin exchange frequency shift.** — Collisions between hydrogen atoms produce the so-called spin exchange frequency shift [19, 20]. With previously considered cavity tuning methods, the related transition frequency offset is largely compensated and affects the maser oscillation frequency only negligibly [21].

The same result can be obtained with the method proposed in this paper, as it is shown in the following. Taking into account spin exchange frequency shifts, the maser oscillation angular frequency ω_1 is given by [22] :

$$\omega_1 - \omega_0 = \left(\frac{1}{T_2'} + \frac{1}{2T_H}\right) \times \times \left[\frac{2(\omega_r - \omega_0}{\delta\omega_c} - C\lambda\right] + \frac{\varepsilon_H}{T_H} \tag{21}$$

where ω_0 is the angular frequency of the atomic transition, in the absence of hydrogen-hydrogen collisions, T_2' is that part of the transverse relaxation time which is not due to H-H collisions, T_H is the contribution of H-H collisions to the longitudinal relaxation time, ω_r has the same meaning as above, $\delta\omega_c$ is the width of the cavity resonance, λ is the spin-exchange frequency shift cross-section and ε_H is a dimensionless parameter which is introduced to represent the effect of the interruption of the oscillating magnetic moments during collisions [20]. C is a constant defined in reference [22].

The quantity T_H^{-1} depends linearly on the atomic density in the bulb situated in the microwave cavity. It can be seen, from equation (21) that it exists a value of ω_r for which the oscillation frequency does not depend on the atomic density. It is given by :

$$2(\omega_r - \omega_0)/\delta\omega_c = C\lambda - 2\,\varepsilon_H \qquad (22)$$

when equation (22) is verified, the angular oscillation frequency takes the value $\omega_1 = \omega_t$ such as :

$$\omega_t - \omega_0 = -\,2\,\varepsilon_H/T_2' \,. \qquad (23)$$

This value of the oscillation frequency is the same as obtained in the frequency method [2, 20, 21], and it is the so-called spin exchange frequency tuned value.

Therefore, the value of ω_r can be adjusted once, and the maser will deliver the proper frequency ω_t.

6. **Conclusion.** — We have described a cavity tuning system of the microwave cavity of hydrogen masers. It has the significant advantage of ensuring a tight control of the cavity and offers a means to improve the long term frequency stability of this frequency standard, and to achieve the ultimate frequency stability determined by the effect of thermal noise on the maser oscillation itself.

APPENDIX

Synchronization properties of a maser. — We are considering the effect of a disturbing signal, with frequency ω, coupled into the microwave cavity of an actively operated maser. We denote $\omega t + \varphi(t)$ the total phase of the perturbed maser oscillation. It can be seen, from basic equations [4] which describes the maser behaviour that we have [14] :

$$\frac{d\varphi}{dt} + \frac{1}{T_2}\frac{p}{b_0}\sin\varphi = \omega_1 - \omega \qquad (24)$$

where T_2 is the transverse relaxation time of the active maser medium, which is related to the line Q factor by

$$T_2 = 2\,Q_l/\omega_1 \qquad (25)$$

p/b_0 is the ratio of the amplitudes, measured in the microwave cavity, of the disturbing and the free-oscillation signals respectively. Equation (24) is valid when the coupled signal perturbs only negligibly the amplitude of the maser oscillation, which is the present situation with $T_2(\omega_1 - \omega) \gg 1$.

Equation (24) is classical in the description of synchronization properties of electronic oscillators [23] or of phase-lock loops [24]. Synchronization occurs for

$$|\,\omega_1 - \omega\,| \leqslant \frac{1}{T_2}\frac{p}{b_0}\,. \qquad (26)$$

This condition is not fulfilled for the values of P_c considered in the text. It can then be shown [24] that the phase $\varphi(t)$ varies periodically and that the beat note between the perturbed oscillator frequency and the externally applied signal has the angular frequency ω_b given by :

$$\omega_b = \left[(\omega_1 - \omega)^2 - \left(\frac{1}{T_2}\frac{p}{b_0}\right)^2\right]^{1/2} \qquad (27)$$

It results from equation (27) that the frequency pulling of the maser is Δf such as :

$$2\,\pi\,\Delta f = \omega - \omega_1 - \omega_b \,. \qquad (28)$$

For $|\,\omega - \omega_1\,| \gg p/T_2\,b_0$, we get :

$$2\,\pi\,\Delta f = \frac{1}{2}\left(\frac{p}{T_2\,b_0}\right)^2 \frac{1}{\omega - \omega_1} \qquad (29)$$

Inserting $(p/b_0)^2 = P_c/P_0$ and $T_2 = 2\,Q_l/\omega_1$, equation (19) is obtained.

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