JOURNAL DE PHYSIQUE
Colloque C8, supplément au n°12, Tome 42, décembre 1981 *page* C8-159

HYDROGEN MASER : ACTIVE OR PASSIVE ?

C. Audoin, J. Viennet and P. Lesage

Laboratoire de l'Horloge Atomique, Equipe de Recherche du C.N.R.S. (*),
Bât. 221, Université Paris-Sud, 91405 Orsay, France

Abstract.- It is shown that, for a specified interrogation scheme of the atomic transition, the cavity pulling factor of passively operated masers is the same as for actively operated ones.
The power spectral density of frequency fluctuations of passive masers is given for a specified atomic line interrogation scheme.
The effect of spin exchange line broadening on the frequency stability capability of passive and active masers is specified. It is shown that it exists an optimum value of the atomic flux intensity. Design criterious are specified.
The frequency stability capability of presently designed small or large size hydrogen masers is compared when operated either actively or passively. It is shown that large size active masers cannot be surpassed, as long as ultimate frequency stability is considered.
The effect of temperature on frequency stability is specified.

I. Introduction-The aim of this paper is to compare the frequency stability capabilities of actively and passively operated hydrogen masers. We will consider

i) Hydrogen masers of classical design with a full size microwave cavity (1). In the following, they will be denoted as "large size hydrogen masers", and

ii) Hydrogen masers with a microwave cavity of reduced size, loaded with a high permittivity dielectric medium (2,3), or with internal capacitors (4,5). They will be labelled as "small size hydrogen masers". Due to the losses of the materials introduced in the cavity, they are operated either as passive devices (2), or as active ones, but with electronically achieved enhancement of the cavity quality factor (6,7).

The reported work has been initiated earlier (8), in the case of passively operated masers. It is completed here by the consideration of the spin exchange line broadening, and of a more realistic frequency modulation scheme for the interrogation of the atomic transition.

In our comparison we will only be interested in the ultimately achievable medium and long term frequency stability. We will then focus on the effect of thermal noise inside the microwave cavity on the white component of the power spectral density of frequency fluctuations. Although introduced when necessary, the effect of noise added by the microwave amplifiers coupled to the microwave cavity is not considered as a fundamental source of frequency stability limitation. This is justified by continual progress in the reduction of the noise figure of these amplifiers.
S.I. Units are used throughout this paper.

(*) Associée à l'Université Paris-Sud

2. Interrogation of the atomic transition in a passive maser.- There are several possible ways to interrogate the atomic transition in a passive maser. We will only consider here the very convenient fast frequency modulation method (2,9), which gives access to the atomic dispersion line.

The frequency modulated microwave signal injected into the maser cavity is represented by the following equation :

$$w_i = p\, e^{j\omega t} \sum_{n=-\infty}^{\infty} J_n(m)\, e^{jn\omega_m t} \qquad (1)$$

where p denotes the amplitude of the signal, ω is the angular frequency of the carrier, ω_m is the modulation angular frequency, m is the frequency modulation index and J_n is the Bessel function of order n.

The amplitude p is defined (8)(10) in such a way that it is not required to specify the coupling factor of the input loop.

At the output port of the maser, the transmitted signal is proportional to :

$$w_t = p\, e^{j\omega t} \sum_{n=-\infty}^{\infty} G^{(n)} J_n(m)\, e^{jn\omega_m t} \qquad (2)$$

where $G^{(n)}$ is the complex gain of the atomic medium, inside the microwave cavity, at frequency $\omega' = \omega + n\omega_m$.

At a given angular frequency ω', this gain G is given by (10) :

$$G = \frac{b}{p'}\, e^{j\varphi} \qquad (3)$$

where b is the amplitude of the atomic response and φ its phase for a sinusoïdal excitation of amplitude p'. We have :

$$G = |G|^2 \left(\frac{p'}{b}\cos\varphi + j\,\frac{p'}{b}\sin\varphi\right) \qquad (4)$$

where $\frac{p'}{b}\cos\varphi$ and $\frac{p'}{b}\sin\varphi$ are given in reference 10, such as :

$$\frac{p'}{b}\sin\varphi = T_c\,(\omega_c - \omega') + \frac{\alpha T_2(\omega_o - \omega')}{1 + T_2^2(\omega_o - \omega')^2 + T_1 T_2 b^2} \qquad (5)$$

$$\frac{p'}{b}\cos\varphi = 1 - \frac{\alpha}{1 + T_2^2(\omega_o - \omega')^2 + T_1 T_2 b^2} \qquad (6)$$

ω_c is the microwave cavity angular resonance frequency, T_c is the microwave cavity time constant, T_1 and T_2 are longitudinal and transverse relaxation times of hydrogen atoms, respectively, and α a parameter, which is unity at oscillation threshold, given, in S.I. units, by :

$$\alpha = \frac{\mu_o \mu_B^2}{\hbar}\, \frac{\eta}{V_c}\, Q_c T_1 T_2 I \qquad (7)$$

where μ_o is the magnetic permeability of vacuum, μ_B is Bohr magneton, $\hbar$ is Planck constant divided by 2π, η is the filling factor, as defined in reference 1, V_c is the volume of the microwave cavity, Q_c is the quality factor of the cavity ($Q_c = \omega_c T_c/2$) and I is the flux of hydrogen atoms entering the bulb in the state $F = 1$, $m_F = 0$.

The signal w_t is amplified, then rectified in a square law detector and finally it is demodulated by multiplication by $\sin \omega_m t$. The error signal V is the d.c. component of the output signal of the demodulator. It is then given by :

$$V = C\, \overline{w_t\, w_t^* \sin \omega_m t} \qquad (8)$$

where C is a constant. The bar means time averaging.

It will be assumed in the following that i) the angular frequency difference $\omega - \omega_o$ is much smaller than the atomic linewidth and ii) the angular frequency modulation ω_m is much larger than the atomic linewidth.

It then comes, from equations 2, 4, 5, 6 and 8 :

$$V = 2Cp^2 J_o(m)\ J_1(m)\ G_o^{\ 2} \frac{\alpha}{1+S_o} \left[T_2(\omega_o - \omega) + T_c(\omega_c - \omega)\right] \tag{9}$$

where G_o and S_o are the gain and the saturation factor respectively, for the carrier component of the interrogation signal of amplitude $p' = pJ_o(m)$ at frequency $\omega' = \omega = \omega_o$.

3. <u>Cavity pulling</u>.- In a steady state regime of the frequency control loop of the quartz crystal oscillator, the error signal V is zero. We then have :

$$P_p = \frac{\omega - \omega_o}{\omega_c - \omega} = \frac{T_c}{T_2} \tag{10}$$

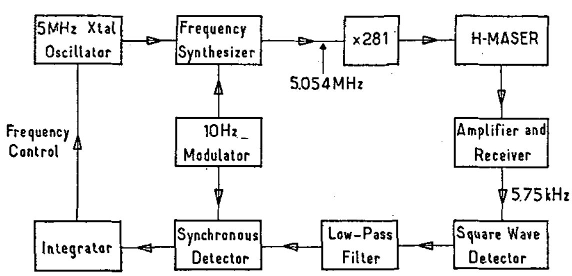
where P_p is the cavity pulling factor of a passive maser, with the specified interrogation scheme.

Equation 10 shows that this cavity pulling factor is the same as for an actively operated maser (1), a result which has been inferred experimentally by F.L. Walls and D.A. Howe (11). The following different result was given earlier (8)

$$P = \frac{T_c}{T_2} \frac{1+S_o}{\alpha} \tag{11}$$

It is valid in different conditions, when the atomic dispersion line is observed using a purely sinusoïdal interrogation signal. This confirms that cavity pulling factors (10) depend on the particular method used to interrogate the atomic transition.

Equation 10 has been checked experimentally. For that purpose, a quartz crystal oscillator has been frequency locked to a passively operated large size hydrogen maser, as depicted on figure 1. The interrogation signal is sinusoïdally frequency modulated.



Experimental set-up for the measurement of the cavity pulling factor of a passively operated H-maser.

Fig.1

Figure 2 shows that the cavity pulling factor has the expected value and does not depend on α, for $S_o \ll 1$. It has also been checked that changes of the saturation factor S_o, and of the modulation index m does not modify the value of P_p.

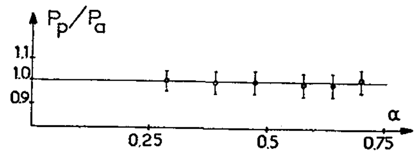


Fig.2

Comparison of the cavity pulling factor P_p of a passively operated maser (with $S_o \ll 1$ and m = 0.1) to the cavity pulling factor P_a of the same maser but operated actively. P_a is set equal to the ratio of separately measured values of T_c and T_2

4. Frequency stability of a passively operated hydrogen maser.-

4.1. Power spectral density of frequency fluctuations.- As previously [8], we assume that thermal noise in the microwave cavity useful mode is created by an appropriate noise generator coupled to a dummy microwave cavity. The output signal then becomes :

$$w_t = \sum_{-\infty}^{\infty} \left\{ G^{(n)} \left[pJ_n(m) + p_2^{(n)} + jp_1^{(n)} \right] + p'^{(n)}_2 + jp'^{(n)}_1 \right\} e^{j(\omega + n\omega_m)t} \qquad (12)$$

where $p_2^{(n)} + jp_1^{(n)}$ represents the complex amplitude of the uncorrelated thermal noise components in the microwave cavity, at frequencies close to $\omega + n\omega_m$. Similarily, $p'^{(n)}_1 + jp'^{(n)}_2$ denotes the additive noise components attached to the microwave receiver.

The one-sided power spectral density (P.S.D.) of these components are given by :

$$\frac{1}{b^2} S_{p_1}(n) = \frac{1}{b^2} S_{p_2}(n) = 4\frac{kT}{P} \qquad (13)$$

and

$$\frac{1}{b^2} S_{p'_1}(n) = \frac{1}{b^2} S_{p'_2}(n) = 4\frac{kT}{P}\frac{Q_{ext}}{Q_c}(F-1) \qquad (14)$$

where k is Boltzman constant, T is the absolute temperature of the cavity, P is the power dissipated in the microwave cavity at frequency $\omega = \omega_o$, Q_{ext} and Q_c are the external and loaded cavity quality factor, respectively, and F is the noise factor of the microwave receiver.

The one-sided P.S.D. of fractional frequency fluctuations $S_y(f) = h_o$ of the controlled quartz crystal oscillator then possesses two components determined by thermal noise in the maser cavity, and by added receiver noise, respectively. It can be shown that we have* :

$$h_o(\text{maser}) = \frac{8kT}{\hbar\omega_o^3}\frac{T_1}{T_2}\frac{(1+S_o)^2}{\alpha^2 S_o} KQ_c C_m \qquad (15)$$

* In reference 8, the contribution of noise at frequencies $\omega \pm 2\omega_m$ is missing.

with

$$C_m = \frac{1}{2J_o^2(m)J_1^2(m)} \left\{ 2J_1^2(m) + \left[J_o(m) - \frac{J_2(m)}{G_o} \right]^2 + \frac{1}{G_o^2} \left[(J_1(m) - J_3(m))^2 + (J_2(m) - J_4(m))^2 + \ldots \right] \right\} \quad (16)$$

and

$$h_o(\text{receiver}) = \frac{8kT}{\hbar\omega_o^3} \frac{T_1}{T_2} \frac{(1+S_o)^2}{\alpha^2 S_o} K Q_{ext}(F-1) C_r \quad (17)$$

with

$$C_r = \frac{1}{2J_o^2(m)J_1^2(m)} \left\{ \left[J_o(m) - \frac{J_2(m)}{G_o} \right]^2 + \frac{1}{G_o^2} \left[2J_1^2(m) + (J_1(m) - J_3(m))^2 + (J_2(m) - J_4(m))^2 + \ldots \right] \right\} \quad (18)$$

In the following, we will focus on the effect of thermal noise in the maser cavity. We will thus determine the ultimate frequency stability capability of a passively operated maser. Expected progress in the noise factor amplifiers justifies the assumption made.

Equation 16 shows that it exists an optimum value m_{opt} of the modulation index m, which depends on G_o. However, it can be shown that for G_o close to unity, we have $m_{opt} \simeq 1.1$ and $C_m \simeq 4.1$ and for the other extreme case $G_o \gg 1$, we have $m_{opt} \simeq 1.1$ and $C_m \simeq 4.3$. We then assume $m = 1.1$ and set $C_m = 4$ in equation 15. We also set $S_o = 1$, which is the optimum value of the saturation factor for $G_o \simeq 1$ and $G_o \gg 1$.

The lower limit of the P.S.D. of a passively operated hydrogen maser is then given by :

$$h_o \simeq 128 \frac{kT}{\hbar\omega_o^3} \frac{T_1}{T_2} \frac{1}{\alpha^2} KQ_c \quad (19)$$

4.2. <u>Effect of spin exchange line broadening</u>.- Relaxation times T_1 and T_2 depend on the atomic beam flux I via H-H collisions in the storage bulb. This effect was first considered in reference 1. We then have :

$$\frac{1}{T_1} = \frac{1}{(T_1)_o} + \frac{2}{T_t} q \frac{I}{I_{th}} \quad (20)$$

$$\frac{1}{T_2} = \frac{1}{(T_2)_o} + \frac{1}{T_t} q \frac{I}{I_{th}} \quad (21)$$

with

$$T_t^2 = (T_1)_o (T_2)_o \quad (22)$$

where $(T_1)_o$ and $(T_2)_o$ are longitudinal and transverse relaxation times for I = 0, respectively. q is the spin exchange maser parameter and I_{th} is the threshold atomic flux for very small values of q. These two quantities are defined as (1) :

$$q = \frac{\sigma \bar{v}_r \hbar}{2\mu_o\mu_B^2} \frac{T_b}{T_t} \frac{1}{V_b} \frac{V_c}{\eta Q_c} \frac{I_{tot}}{I} \quad (23)$$

$$I_{th} = \frac{\hbar}{\mu_o\mu_B^2} \frac{V_c}{\eta Q_c} \frac{1}{T_t^2} \quad (24)$$

σ is the spin exchange cross-section, $\overline{v_r}$ is the average relative hydrogen velocity, T_b is the bulb storage time constant, V_b is the storage bulb volume and I_{tot} is the total flux entering the bulb.

It comes, from equations 7, 19, 20 and 21 :

$$h_o \simeq \frac{8kT}{K^2 \omega_o^3} \mu_o \mu_B^2 \frac{\eta Q_c}{V_c} H_p(q, \frac{I}{I_{th}}) \qquad (25)$$

with

$$H_p(q, \frac{I}{I_{th}}) = 16 \frac{1+q\frac{I}{I_{th}}}{1+2q\frac{I}{I_{th}}} \left[1+3q\frac{I}{I_{th}} + 2(q\frac{I}{I_{th}})^2\right] \frac{1}{(I/I_{th})^2} \qquad (26)$$

Figure 3 shows the variation of H_p versus I/I_{th} for different values of the parameter q. One sees that for each value of q, there is a value of the atomic flux which minimizes the P.S.D. of fractional frequency fluctuations. The locus of the minimum of H_p occurs for an optimum value of I/I_{th} given by the following equation :

$$\frac{I}{I_{th}} \text{opt} \simeq \frac{0.845}{q} \qquad (27)$$

and we have :

$$H_p(\text{min.}) \simeq 379\, q^2 \qquad (28)$$

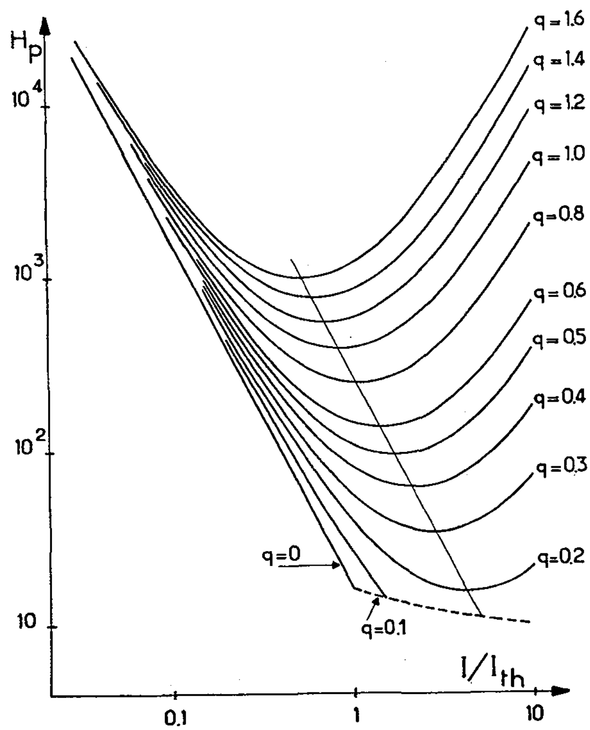


Fig.3

Variation of $H_p(q, I/I_{th})$ versus I/I_{th} for different values of the parameter q. The dotted line represents the threshold condition of oscillation and the thin line represents the locus of the minima of H_p.

At the minimum of H_p, one also has :

$$\alpha \simeq 0.17/q \tag{29}$$

It comes from equations 23, 25 and 28 :

$$h_o(\text{min}) \simeq 758 \frac{kT}{\mu_o \omega_o^3} \left[\frac{\sigma \overline{v}_r}{\mu_B} \frac{T_b}{T_t} \frac{I_{tot}}{I}\right]^2 \frac{1}{V_b^2} \frac{V_c}{\eta Q_c} \tag{30}$$

The design goal of passively operated hydrogen masers is then to make the quantity $V_c/\eta Q_c V_b^2 \propto q/V_b$ as small as possible.

In addition, it can be derived, from equations 23, 24 and 27 that the optimum value of the atomic flux is the following :

$$I(\text{opt.}) \simeq 1.69 \frac{V_b}{\sigma \overline{v}_r} \frac{I}{I_{tot}} \frac{1}{T_b T_t} \tag{31}$$

Table 1 shows the ultimate frequency stability capability of presently designed passive hydrogen masers.It is denoted $\sigma_y(\tau)$, and calculated for τ = 100 s.

	Large size maser		Small size maser (passive)	
	Active	Passive	Cavity loaded with alumina*	Cavity loaded with capacitors
V_c	15.5×10^{-3}	15.5×10^{-3}	2.3×10^{-3}	2.35×10^{-3}
V_b	2.35×10^{-3}	2.35×10^{-3}	1.15×10^{-3}	0.93×10^{-3}
η	2.8	2.8	0.5	(η'=0.5) 1.25
Q_o	60 000	60 000	6 000	13 000
Q_c	45 000	30 000	3 000	6 500
$\eta Q_c/V_c$	8.1×10^6	5.4×10^6	6.5×10^5	3.4×10^6
q	0.058	0.087	1.44	0.34
T_t	0.4	0.4	0.2	0.2
I_{th}	7.5×10^{11}	1.1×10^{12}	3.7×10^{13}	7×10^{12}
I/I_{th}(opt)	15.4	($\alpha \simeq 1$) 1.4	0.59	2.5
I(opt)	1.1×10^{13}	1.5×10^{12}	2.2×10^{13}	1.7×10^{13}
H_a(opt)	0.35	-	-	-
H_p(opt)	-	15	786	44
h_o	1.4×10^{-27}	3.8×10^{-26}	2.4×10^{-25}	7.1×10^{-26}
$\sigma_y(\tau = 100$ s)	2.6×10^{-15}	1.4×10^{-14}	3.5×10^{-14}	1.9×10^{-14}
$\sigma'_y(\tau = 100$ s)	2.6×10^{15}	2.4×10^{-14}	6×10^{-14}	3.2×10^{-14}

TABLE 1. A comparison between actively and passively operated masers.

*With a sapphire loaded cavity [3], with Q_c = 8 500, it comes $\sigma_y(\tau = 100$ s) = 2.1×10^{-14}

5. Frequency stability of an actively operated hydrogen maser. - The one-sided P.S.D. of fractional frequency fluctuations of an actively operated hydrogen maser is given, for Fourier frequencies f smaller than the microwave cavity bandwidth by [12] [13] [14] :

$$S_y(f) = \frac{4kT}{P}\left\{\frac{1}{4Q_\ell^2} + \left[1 + (F-1)\frac{Q_{ext}}{Q_c}\right]\frac{f^2}{\nu_0^2}\right\} \tag{32}$$

where $Q_\ell = \omega_0 T_2/2$ and ν_0 is the transition frequency.

We will only consider h_0, the white frequency component of $S_y(f)$, which determines the medium term frequency stability* and, hopefully [15] long term frequency stability in the absence of drift in the cavity frequency. One then has :

$$h_0 = \frac{4kT}{P}\frac{1}{\omega_0^2 T_2^2} \tag{33}$$

The variation of P versus I/I_{th} is given in reference 1 as :

$$\frac{P}{P_c} = -2\left(q\frac{I}{I_{th}}\right)^2 + (1-3q)\frac{I}{I_{th}} - 1 \tag{34}$$

with

$$P_c = \frac{1}{2}\hbar\omega_0 I_{th} \tag{35}$$

It results from equations 21, 33 and 34, that the effect of spin exchange line broadening [16] upon the value of h_0 can be written as :

$$h_0 = \frac{8kT}{\hbar^2\omega_0^3}\mu_0\mu_B^2\frac{\eta Q_c}{V_c}H_a\left(q, \frac{I}{I_{th}}\right) \tag{36}$$

with

$$H_a\left(q, \frac{I}{I_{th}}\right) = \frac{\left(1 + q\frac{I}{I_{th}}\right)^2}{\left[-2\left(q\frac{I}{I_{th}}\right)^2 + (1-3q)\frac{I}{I_{th}} - 1\right]} \tag{37}$$

Figure 4 shows the variation of H_a versus I/I_{th} for different values of the parameter q. Again, for each value of $q \neq 0$, there is a value of the atomic flux which minimizes the P.S.D. of fractional frequency fluctuations. The locus of the minimum of H_a is defined such as :

$$\frac{I}{I_{th}}(\text{opt}) = \frac{1}{q}\frac{1-q}{1+q} \tag{38}$$

and we have :

$$H_a(\text{min.}) = \frac{4q}{q^2 - 6q + 1} \tag{39}$$

*It is with noticing that h_0 does not depend on the receiver noise factor, for actively operated masers.

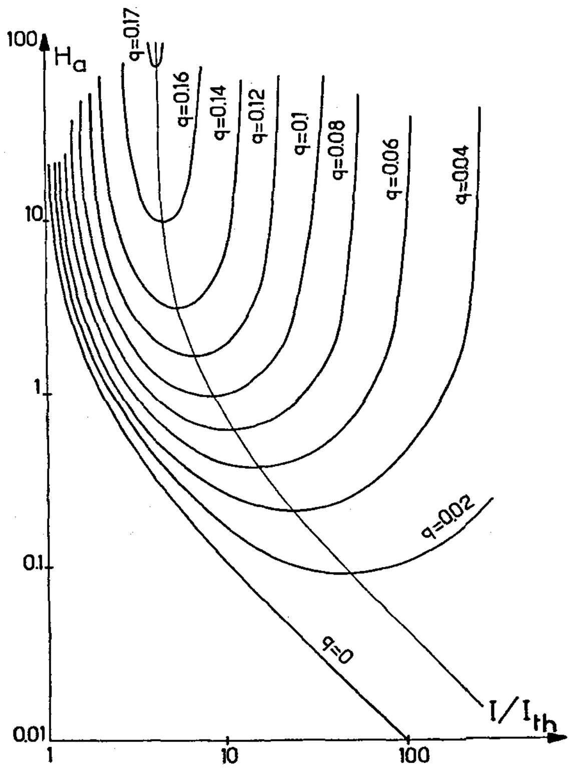


Fig.4

Variation of $H_a(q, I/I_{th})$ versus I/I_{th} for different values of the parameter q. The thin line represents the locus of the minima of H_a

It comes, with equations 23, 36 and 39 :

$$h_o(\text{min.}) = 16\,\frac{kT}{\hbar\omega_o^3}\,\sigma\,\overline{v}_r\,\frac{T_b}{T_t}\,\frac{I_{tot}}{I}\,\frac{1}{V_b}\,\frac{1}{q^2-6q+1} \tag{40}$$

This equation shows that the white frequency noise of actively operated masers is small if V_b is large* and q as small as possible. However, the change of h_o(min) with q is small as long as $q < 0.05$.

The optimum value of the atomic flux is given by :

$$I(\text{opt.}) = 2\,\frac{V_b}{\sigma\overline{v}_r}\,\frac{I}{I_{tot}}\,\frac{1}{T_bT_t}\,\frac{1-q}{1+q} \tag{41}$$

where the quantity 1-q/1+q does not change much in the range $0 < q < 0.172$ in which the maser is able to oscillate [1].

Table 1 shows the related frequency stability capability of a large size maser of classical design.

* One should notice the interest of the elongated bulb and cavity design by H.E. Peters [17], which enables to increase the bulb volume.

6. A comparison of the frequency stability capability of actively and passively operated masers.- We apply the above results to establish a comparison between the ultimate medium and long term frequency stability capabilities $\sigma_y(\tau) = (h_o/2\tau)^{1/2}$ of some presently designed hydrogen masers.

We consider

i) a large size hydrogen maser in which only a hydrogen storage bulb (with a diameter of about 16.5 cm) is included in the microwave cavity ; we will assume this maser either actively, or passively operated (but very close to oscillation threshold in the second case),

ii) small size hydrogen maser with a microwave cavity loaded either by alumina [2] or by internal capacitors [5].

We assume that we have : $\sigma = 23.5 \times 10^{-20}\ m^2$ at room temperature [18] [19], $T_b/T_t = 1.3$, $I_{tot}/I = 2$ and that the output loop is critically coupled for passive masers.

Table 1 shows the pertinent parameters and the expected ultimate frequency stability measure σ_y for $\tau = 100$ s. Receiver noise degrades the considered frequency stability of passive masers only. The frequency stability capability figure σ'_y, given in Table 1, is then obtained for $F = 2$ and $Q_{ext} = 2Q_c$.

The following conclusions can be made :

i) experimentally measured frequency stability of actively operated hydrogen masers is close (within a factor of 2) to the ultimate frequency stability capability in the white frequency noise region [13] [14] ; the same conclusion can be derived for the white phase noise region.

ii) except for laboratory testing, there is no interest to operate passively, a large size H-maser, within condition $\alpha < 1$.

iii) measured frequency stability of passive small size masers [20] is also close (within a factor of 2) to the ultimate frequency stability capability,

iv) the optimum value of the atomic flux intensity is slightly larger for small size passive masers than for large size active masers and

v) presently designed small size passive masers cannot compete with large size masers, as long as frequency stability is concerned.

The value of the spin-exchange parameter q varying as $\sigma \overline{v_r}$, this value will be drastically reduced [18] in low temperature masers [16] [21] so that small size hydrogen masers will very likely be able to oscillate in such conditions. Equation 40 shows that for $q \lesssim 0.05$, the frequency stability capability $\sigma_y(\tau)$ of an active maser scales as $T^{3/4}(\sigma/V_b)^{1/2}$. One may then expect a spectacular frequency stability improvement of 2 or 3 orders of magnitude at low temperature. Possible frequency stability limitation by quantum noise should then be considered.

7. Effect of cavity quality factor enhancement on the frequency stability.- Small size hydrogen masers have been operated actively by enhancing artificially [6] [7], by electronic means, the cavity quality factor. The question then arises of their frequency stability capability compared to that of the same device, but operated passively.

Quality factor enhancement increases the noise temperature T'_e of the cavity when a feedback loop is used, we have [22] :

$$T'_e = \frac{Q_e}{Q_c} T \left[1 + \beta_1 (F-1)\ G^2 \frac{Q_c}{Q_o}\right] \tag{42}$$

where Q_e and Q_o are the enhanced and unloaded cavity quality factors, respectively, β_1 is the coupling factor of the loop connected to the input of the microwave amplifier, G is the gain introduced to achieve the value Q_e of the quality factor and F is the noise figure of the amplifier. It has been assumed that the microwave cavity and the amplifier are at the same temperature.

Neglecting the noise added by electronic components which provide Q-enhancement, the cavity noise temperature becomes T_e given by :

$$T_e = \frac{Q_e}{Q_c} T \quad (43)$$

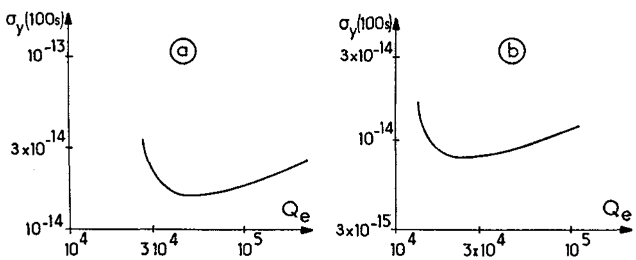
Equation 43 merely means that thermal energy of the cavity mode being distributed over a smaller bandwidth, its P.S.D. is increased accordingly.

It is easy to see that equation 36 then becomes :

$$h_o = \frac{8kT}{\hbar^2 \omega_o^3} \mu_o \mu_B^2 \frac{\eta Q_c}{V_c} \left(\frac{Q_e}{Q_c}\right)^2 H_{a,e}\left(q, \frac{I}{I_{th}}\right) \quad (44)$$

where $H_{a,e}$ is given by equation 37, but with the values of q and I_{th} which are related to Q_e.

Equation 44 has been applied to presently designed small size hydrogen masers, with characteristic parameters as given in Table 1, except for the cavity quality factor : one assumes now that the cavity is coupled to external circuits with two loops having coupling factors $\beta_1 = \beta_2 = 0.2$. Figure 5 shows that there is an optimum value of Q_e.



Ultimate frequency stability capability σ_y, for $\tau = 100$ s of presently designed small size masers, versus the enhanced cavity quality factor Q_e.

a. microwave cavity loaded with aluminia
b. microwave cavity loaded with capacitors.

Fig.5

Table 2 summarizes the results. It includes the value of the frequency stability measure σ'_y (100 s) when the effect of noise of the amplifier, included in the feedback loop (cf. equation 42), is taken into account with F = 2.

	aluminia loaded cavity	capacitor loaded cavity
Q_c	4 300	9 400
Q_e(opt)	50 000	25 000
σ_y(100 s)	1.6×10^{-14}	7.8×10^{-15}
σ'_y(100s)	2.6×10^{-14}	1.1×10^{-14}

TABLE 2. Frequency stability capability of small size hydrogen masers operated actively with enhanced cavity quality factor.

Comparison of results given in Tables 1 and 2 shows that one may expect an improvement (by a factor of 2 to 3) in the ultimate frequency stability of small size hydrogen masers when operated actively via Q-enhancement. However, the cavity pulling factor will be increased accordingly.

8. Conclusion.- The major conclusion of this work is that large size active masers remain , at present, the atomic frequency standardsof the best frequency stability capability. One often object against their relatively poor long term frequency stability. However, no serious attempt has been made to operate an efficient cavity auto-tuning system. It has been shown both theoretically [15] and experimentally [6] that the best solution of this problem would be to use a fast auto-tuning system similar to that associated with passive masers. One may then expect that the fractional frequency stability measure $\sigma_y(\tau) = (h_o/2\tau)^{1/2}$, with h_o given by equation 40 will be achieved for very large values of τ, larger than several days.

The frequency stability improvement which may be expected from low temperature masers has been pointed out.

References

(1) KLEPPNER D., BERG H.C., CRAMPTON S.B., RAMSEY N.F., VESSOT R.F.C., PETERS H.E. VANIER J., Phys. Rev. A 138 (1965) 972
(2) HOWE D.A., WALLS F.L., BELL H.E. and HELLWIG H., Proceedings of the 33rd Annual Symposium on Frequency Control p. 554. Electronic Industries Association Washington DC, 1979
(3) MATTISON E.M., BLOMBERG E.L., NYSTROM G.U. and VESSOT R.F.C., Proceedings of the 33rd Annual Symposium on Frequency Control, p. 549. Electronic Industries Association, Washington DC,1979
(4) PETERS H.E., Proceedings of the 32nd Annual Symposium on Frequency Control p. 469. Electronic Industries Association Washington DC, 1978
(5) WANG H.T.M., LEWIS J.B. and CRAMPTON S.B., Proceedings of the 33rd Annual Symposium on Frequency Control p. 543. Electronic Industries Association, Washington DC,1979
(6) WANG H.T.M., Proceedings of the 34th Annual Symposium on Frequency Control, p. 365. Electronic Industries Association, Washington DC, 1980
(7) PETERS H.E., Proceedings of the 35th Annual Symposium on Frequency Control, 1980. To be published
(8) LESAGE P., AUDOIN C. and TETU M., Proceedings of the 33rd Annual Symposium on Frequency Control, p. 515. Electronic Industries Association, Washington DC, 1979
(9) BUSCA G., Proceedings of the 33rd Annual Symposium on Frequency Control, p. 563 Electronic Industries Association, Washington DC, 1979
(10) VIENNET J., AUDOIN C. and DESAINTFUSCIEN M., IEEE Trans. on Instr. and Meas. IM-21 (1972) 204
(11) WALLS F.L. and HOWE D.A., Conference on Precision Electromagnetic Measurements. Braunschweig (F.R.G.) June 1980
(12) CUTLER L.S. and SEARLE C.L., Proc. of the IEEE 54 (1966) 136
(13) VESSOT R.F.C., Final Report Contract JPL 95 4818; Smithsonian Institution Astrophysical Observatory, Cambridge USA, 1977
(14) LESAGE P., Thèse de Doctorat d'Etat - Université Paris-Sud, Orsay 1980
(15) AUDOIN C., Revue de Physique Appliquée 16 (1981) 125
(16) VESSOT R.F.C., LEVINE M.W. and MATTISON E.M., Proceedings of the 9th Annual Precise Time and Time Interval p. 549, Goddard Space Flight Center, Greenbelt USA, 1977
(17) PETERS H.E., Mc GUNIGAL T.E. and JOHNSON E.H., Proceedings of the 22nd Annual Symposium on Frequency Control p. 464. National Technical Information Service, Springfield USA, 1968
(18) ALLISON A.C., Phys. Rev.A 5 (1972) 2695
(19) DESAINTFUSCIEN M. and AUDOIN C., Phys. Rev. A 13 (1976) 2070
(20) WALLS F.L.. This issue
(21) CRAMPTON S.B., SOUZA S.A. and KRUPCZAK G. This issue
(22) LESAGE P. and AUDOIN C., IEEE Trans. on Instr. and Meas. IM-30 (1981) 182